Investigation **1**

Deflection of a Center-Loaded Rectangular Beam

Imagine you are walking across a simple plank that spans a creek and you stop in the middle of the plank to enjoy the scenery. Consider the plank you are standing on. You will notice it is bowing a bit downward where you stand. If you were to change the dimensions, could you make the beam less flexible (more rigid)?

PRELIMINARY OBSERVATIONS

1. Find a small uniform beam, such as a Popsicle stick or a pencil. Place your fingers on the ends and gently push up on the center with your thumbs. What do you observe?

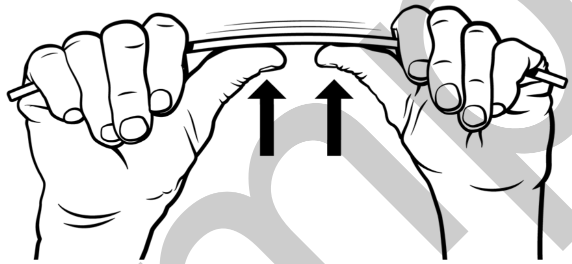

Figure 1 Use one or both thumbs to apply a force in the middle of the beam

2. Discuss your observations with your group or class. Consider the following questions:
   - How many beam characteristics can you identify that may affect the beam's flexibility?
   - What dimensions of the beam could you change to make the beam more stiff?
   - Which dimension will have the greatest effect on the stiffness of the beam?
   - Can you identify any changes that will not affect the stiffness of the beam?
   - Draw a qualitative graph that represents the relationship between force and deflection at the center of the beam. Label the axes and explain your reasoning.
3. Observe as your instructor places a beam on the Structures & Materials Tester and applies a load to the center of the beam. Notice that the beam is centered over the force sensor and the cross-bars are equal-distance from the center of the apparatus. Also note when and how to zero the force and displacement sensors so that when you are using the equipment on your own, you will be able to set it up correctly.

INVESTIGATION

You will investigate one or more elements that affect the flexibility of the beam:

1. Develop a procedure for your investigation.
   - Include the question you will investigate.
   - Articulate your hypothesis, and, if appropriate, propose a relationship between variables.
   - List the measurement equipment you will use.
   - Decide how much data to take in order to have enough information to satisfy your purpose and stand up to questioning by your peers.
   - Remember to change only one independent variable at a time.
2. Discuss and decide which factors you will measure to develop your model of the flexibility.
3. Carry out the investigation.

ANALYSIS

For each part of the procedure consider the following questions: Is the graph of variables you measured a linear graph? If not, you may need to perform one or more mathematical operations to linearize your data or use the curve fit tool to determine the mathematical relationship. Develop a mathematical model for your data and discuss with your group how your variables fit into the model. When you discuss the results with your class, be sure to share your model and ideas. You may also wish to do some research to support your experimental data.

EXTENSION

Do you think there is a difference between using a plank that is 2"×4" on edge and using two 2"×2" planks stacked (but not secured together). Test this idea by creating samples appropriate for the Structures & Materials Tester.

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INTRODUCTION

The first four investigations in this book follow a guided-inquiry format and lead up to the final activity in which students are challenged to design the most efficient bridge based on the knowledge they have gained through investigation. Each investigation begins with Preliminary Observations, which are designed to engage the student physically and mentally with the phenomena. After facilitating a class discussion about students' observations, you will pose a question to the students that frames the intended investigation. Following this structure allows you to assess students' pre-existing knowledge and assumptions as well as offers a way to engage the students in the process of designing their investigation.

Refer to the Tips section for additional recommendations regarding the demonstrations for the Preliminary Observations in each investigation. Also note that the Preface of this book contains helpful information about how to set up and teach with the Go Direct Structures & Materials Tester and the Graphical Analysis 4 data-collection app.

OVERVIEW OF INVESTIGATION 1

Students should finish this activity with a clear model of the factors affecting the deflection of a beam. In the Preliminary Observations, students observe a beam bending under a load and brainstorm what physical characteristics affect the amount of bending (aka deflection) that occurs. Students should ultimately use their model to predict the deflection of a beam, given specific beam parameters and a known applied force.

WHAT SHOULD STUDENTS KNOW BEFORE DOING THIS ACTIVITY?

Students should have an intuitive grasp of the individual factors that affect the flexibility of a beam. A class discussion should reveal the general nature of the individual relationship. For example, the greater the span between the supports, the greater the beam deflects with a given load. Students need not be familiar with the modulus of elasticity (E), but will recognize that the type of material will also affect the deflection.

INSTRUCTOR INFORMATION

This activity offers students the opportunity to explore the factors affecting the flexibility of a center-loaded beam that is supported at both ends. The student handout directs students to push up on a beam that is supported on both ends and observe the resulting deflection.

The equation that models the deflection behavior is

$$\Delta S = \frac{FL^3}{48EI}$$

where Δs is the beam's elastic, vertical displacement at mid-span, F is the load, L is the span length, E is the modulus of elasticity, and I is the area moment of inertia. If we consider a solid rectangular beam of base b and height h, then the area moment of inertia, I, is $bh^3/12$, and the model becomes

$$\Delta S = \frac{FL^3}{4Ebh^3}$$

If students express uncertainty relating to the general nature (direct or inverse) of the factors, a discussion of extremes may help. For example, compare the amount of bending that occurs when you stand on a 2×4 that spans a short length versus one that spans 10 times the length. This should allow them to derive the relative relationships as shown in this (incorrect) proportionality:

$$\Delta \propto FL/bh$$

This is a good starting point for students to use as a prediction of what they might expect from their investigation. When this preliminary model breaks down, it will be important to encourage students to stop and evaluate the system and hypothesize why the data show what they do.

LEARNING OUTCOMES

- Students will engage in an inquiry activity to develop a mathematical model based on observations (of their own devising) of the physical world.
- Students will understand the factors that affect the deflection of a center-loaded rectangular beam, supported on both ends, including width, thickness, and length.
- Students will construct a model representing the relationship of factors affecting the deflection of center-loaded rectangular beams.

NEXT GENERATION SCIENCE STANDARDS

Disciplinary Core Ideas	Crosscutting Concepts	Science and Engineering Practices
ETS1.A. Defining and Delimiting Engineering Problems ETS1.B. Developing Possible Solutions ETS1.C. Optimizing the Design Solution	Patterns Cause and effect Scale, proportion, and quantity Systems and system models	Asking questions and defining problems Developing and using models Planning and carrying out investigations Analyzing and interpreting data Using mathematics and computational thinking Constructing explanations and designing solutions Engaging in argument from evidence

ESTIMATED TIME

The project is designed to be completed in two 45-minute periods, or less if you break the activity into individual assignments for investigation and then bring the parts together in a class discussion. In the latter approach, plan on a single 45-minute period, with the potential need for an additional 20 minutes on a following day to integrate all the components into your students' models.

MATERIALS

Make the following materials available for student use. Items in bold are needed for the Preliminary Observations.

computer, Chromebook, or mobile device
Graphical Analysis 4 app
Go Direct Structures & Materials Tester (GDX-VSMT)
rectangular beam
rulers or Popsicle sticks
rectangular beams of adequate number (5 different widths and thicknesses; see Tips for materials recommendations)

PRELIMINARY OBSERVATIONS

In the Preliminary Observations, students are instructed to take a simple rectangular beam, such as a ruler or Popsicle stick, and test it for bending while thinking about the characteristics that may affect its flexibility.

Question the students to help them think about how changing a characteristic of the beam will affect the flexibility. For example, "If you increase the height of the beam, will it bend more or less or perhaps not change at all?" is an appropriate initial probe. You may follow up by asking them, "If you double the height, how will the deflection be affected?"

IMPLEMENTATION

After the Preliminary Observations and discussion, frame the investigation. Challenge students to identify beam characteristics that affect the flexibility of the beam. You can decide to have each lab group choose one characteristic to investigate, or have all lab groups evaluate all characteristics. If you choose the former approach, we recommend bringing the students back together as a class to share their results and build a more comprehensive model.

Demonstrate the general approach to using the Structures & Materials Tester by attaching appropriate tackle (e.g., eye-bolt, U-bolt, and quick link, if needed) and placing a beam on the cross bars. Demonstrate how the equipment can be used to gather force and displacement data. Show students how to zero the sensors in Graphical Analysis before starting data collection. This is especially important for the displacement sensor, as the zero point changes with samples of different heights.

Provide students with the materials for the investigation. We recommend the use of basswood dimensioned samples, which can be purchased from several online sources or local hobby shops. You can purchase longer lengths and cut to an appropriate length (~30 cm works well).

ANALYSIS

Students should notice a direct relationship between span and displacement, meaning that as the span increases the vertical displacement also increases. They should also observe an indirect relationship between both width and thickness as it relates to displacement.

Students should either linearize the data by creating a calculated column or use the curve fitting tools to determine the mathematical relationship between displacement and each variable.

The resulting model should (ideally) be in the form: $\Delta = AFL^3/bh^3$. Where F is the force applied, L is the span, b is the base width, and h is the height of the beam. The parameter A is a constant of proportionality that incorporates the modulus of elasticity (E). This attribute will be explored in more detail in Investigation 3 of this book. Students can enter all of the values for a given beam under a certain load and then algebraically determine the value of A.

SAMPLE RESULTS

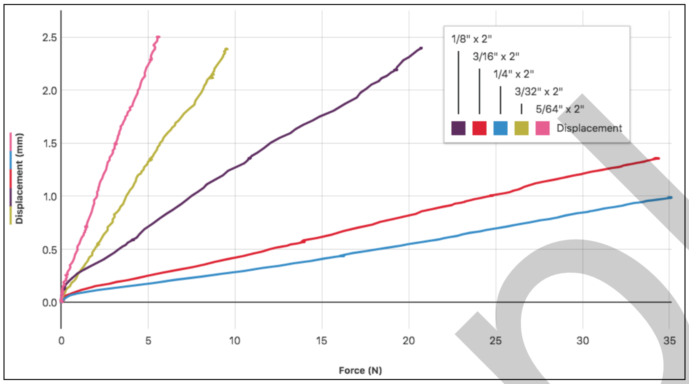

Figure 1 Five data sets of displacement versus force

This data set may or may not be part of the students' evaluation since it is not a variable of the beam. However, it is interesting to note that the relationship is proportional and that the beam behavior is "Hookian" (follows Hooke's Law) in the ranges of forces tested.

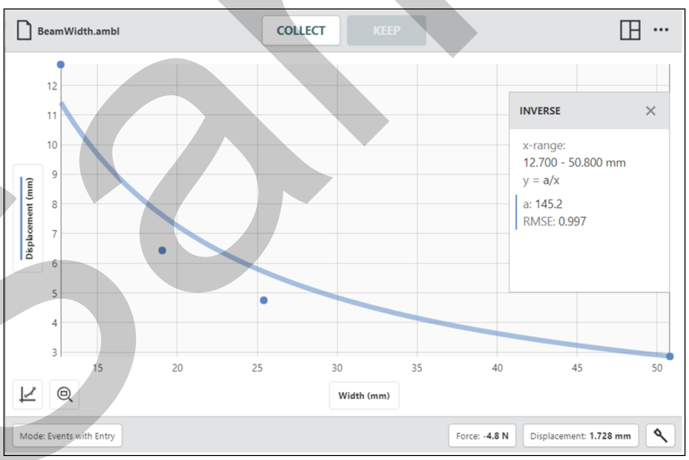

Figure 2 Five data points with an inverse curve fit showing the relationship between beam width and displacement

The power curve fit for the data shows an inverse relationship between beam width and displacement. Note that the beam length and height are kept constant during this investigation.

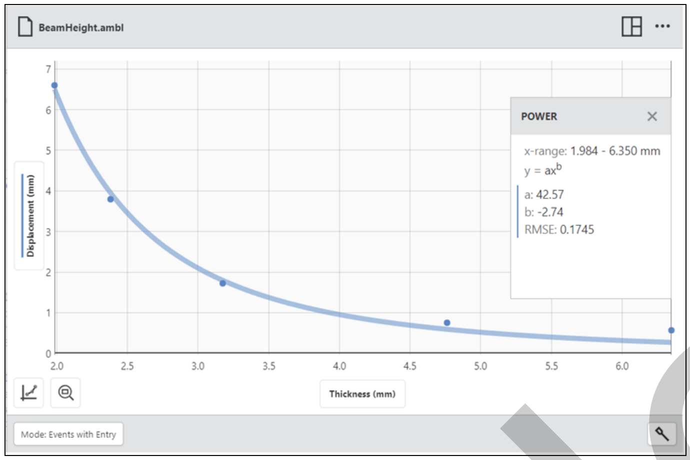

Figure 3 Beam height versus displacement

Students may not have previously encountered an inverse-cubed relationship.

Figure 4 Span versus displacement

The span between the supports of the beam and the resulting displacement results in a cubed relationship.

TIPS

1. Follow these safety recommendations found in the Go Direct Structures & Materials Tester *User Manual* for additional best practices:
   - Wear safety glasses.
   - Any tackle connected to the Structures & Materials Tester using threaded parts should be attached so that a sufficient amount of the threaded component is engaged.
   - Quick links should be secured and not left open.

2. Basswood is generally preferred over balsa wood. Balsa wood tends to be soft and brittle, can break under relatively small forces, and contains a large degree of variation.

3. Thin pieces of wood work best. All experiments can be done with wood with a thickness of 0.25 in (5 mm) or less. Using thin pieces will also allow you to keep the load forces smaller so that not as much energy is released in the event of beam failure.

4. Although not included in the materials list, a precision measurement device, such as a Vernier caliper or micrometer, can be used to more accurately measure the dimensions of the beams.

5. In this activity, there is no need to apply forces that will bend the beams to the breaking point. The beams can be used in multiple activities.

6. The experiments described in this activity make use of a U-bolt and quick link (found in the Structures & Materials Tester tackle kit) to securely connect the beams to the device. It is not necessary to consider the weight of this tackle in the calculations of force. The added weight will affect the intercept of the equation, but not the relationship between force and displacement.

EXTENSION

Students should find that stacking individual beams gives an inverse square relationship, rather than an inverse cubed relationship observed for solid beams. This illustrates the importance of the internal forces acting on a center loaded beam.

